LOGICAL, SCIENTIFIC AND REAL POSSIBILITY

1. Real vs. abstract possibility

In the course of the history of logic and natural philosophy, the modalities of possibility/necessity have been explicated e.g. by means of the notion of *truth* and its *temporal variability* vis-à-vis *stability* with respect to propositions. Necessity has been associated with preserving a truth-value, and possibility has been associated with truth at some moment in time. Propositions that have their truth-values constant have been considered *necessary,* while those that have their truth-values variable (being true at some moment in time and false at another) have been considered *possible.* These notions of necessity and possibility are, of course, distinct from logical necessity and logical possibility.

Since the course of events is determined by the laws of nature, the Stoics (Chrysippos) purported to define necessity/possibility not only with respect to *generally* (logically?) *possible*, but also to the *laws of nature* and *external circumstances*—which can be regarded as the empirical modalities of *real possibility* and *real necessity.* It is here that we can discern one of the first attempts to explicate the common notions of real and abstract possibility, or real impossibility.

The Stoics defined the modalities as follows:

'A proposition is possible which admits of being true, there being nothing in external circumstances to prevent it being true, *e.g.* "Diocles is alive". Impossible is one which does not admit of being true, as *e.g.*: The earth flies". That is necessary which besides being true does not admit of being false or, while it may admit of being false, is prevented from being false by circumstances external to itself, as "Virtue is beneficial". Not necessary is that which, while true, yet is capable of being false if there are no external conditions to prevent, e.g. "Dion is walking". A reasonable proposition is one which has to start with more chances of being true than not, e.g. "I shall be alive tomorrow".' [DL VII, 75]

We shall return to the analysis of this conception later.

2. Criticism of the 'standard' theory of modalities

The prevalent or "standard" explication of modalities offered by present-day logic—the one based on C. I. Lewis theory and Kripke semantics—has been radically criticized by P. Tichy. He points out its primarily formalistic rather than contentual approach: the „box" sign (□)—a logician's symbol for necessity—is ambiguous and has only been defined implicitly.

All the systems S1–S5 use the same symbols—the box (□) for necessity and the diamond (◊) for possibility—although the notions attached to them in the respective axiomatizations differ due to the different formal properties of the accessibility relation. Kripke's purported definition of a necessary proposition, defined as one which is true in all accessible worlds, suffers from a serious defect: the accessibility relation has not been defined, though this notion cannot be taken as familiar or intuitive. Even so, only the formal properties of this mysterious relation of accessibility between possible worlds are to determine a particular meaning of the necessity operator.

> Thus, although Kripke´s theory is spoken of as a *semantics* for modal logics, it does not begin to throw light on the meaning of the box. It simply trades one undefined notion for another. [Tichý 1988, 279].

There is an additional difficulty as to which type the necessity operator is. This is clearly shown even by e.g. Sainsbury's approving analysis [2001, 304–317].

3. Logical vs. empirical modalities

The crucial fact about the notions of necessity and possibility as explicated by S1–S5 (including other 'intermediary' systems) is that the sentences of the form □A and ◊A are interpreted as analytical, non-empirical sentences (either true or false), or as non-analytical sentences whose truth depends on empirically ascertainable states of affairs—the possible worlds.

Let us consider 'paradigmatic' propositions expressed by the following sentences and examine what type of modalities they contain:

1. It is necessary that $5 + 7 = 12$.

2. It is necessary that marble statues are marble.

3. It is possible that Socrates is a prime number.

4. It would be possible that Socrates was black.

5. If Socrates had not been a philosopher, he could have possibly been a warleader.

6. It was possible that the Persians would defeat the Greeks at Marathon in 490 BC.

7. It is possible that it will rain tomorrow.

8. It is possible that it will not rain tomorrow.

9. It is necessary that it will rain tomorrow.

10. It was necessary that Greeks defeated Persians at Marathon in 490 BC.

11. If ice is put in water, it will necessarily float.

12. The speeds of fall of two bodies are necessarily equal.

13. The speed of fall of a body is necessarily dependent upon (the gravitational constant and) the elapsed time only.

14. The speed of fall of a body is dependent upon on its shape and the air resistance.

It seems that some of these are analytical—undoubtedly such is the proposition expressed by the sentence (1), which is a mathematical fact independent of empirical circumstances. The same could probably be said about the sentence[1] (2) at first sight, since many people take it for a logical fact. The sentence (3) is logically absurd—it is logically inconceivable for (3) to become true (supposing the standard use of the expression *Socrates* as a proper name denoting the famous philosopher).

On the contrary, we do not take the sentence (4) as being true independently of the actual state of affairs, as it is logically conceivable (provided we are not extreme essentialists and do not take the essence of Socrates consist of the properties of being white, being a philosopher, etc.) that Socrates could have been born black or that he might have become a sculptor or warleader, had he not become a philosopher. We assume that no laws of nature blocked this possibility.

The sentence (5) is a counterfactual conditional whose consequent refers to an alternative not actualised in the real world. However, by saying the sentence (5) we want to say more than just its logical conceivability, viz.: it was a *real* possibility that, had some relevant events happened and had others happened not, Socrates would become e.g. a warleader—we assume that no laws of nature or external circumstances blocked this possibility. Therefore, we do not speak of possibility of a purely logical kind here. A similar situation is with the sentence (6).

The sentence (7) seems at the first glance to be true regardless of circumstances, i.e. an analytical truth, since surely it will rain sometime. This is a rule in the actual world, yet it is an empirical generalization. Undoubtedly there are conceivable worlds in which it never

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[1] To be true or false, and analytical or empirical are, strictly speaking, properties of propositions which are expressed by sentences. For simplicity's sake we shall feign to assign these properties directly to sentences in the spirit of the established tradition and of Church's note in *Introduction to Mathematical Logic*, §04, 27 that we can assert both a proposition and a sentence expressing it. (When a sentence expressing a proposition is asserted we shall say that the proposition itself is thereby *asserted*).

rains. Thus, the sentence (7) expresses an empirical proposition, and the modality it contains is not a logical one.

If (7) expresses an empirical proposition, then (8) is *a fortiori* empirical. Surely, there exist days in the actual world followed by rainy days as well as those followed by days without rain. Therefore, its truth-value depends on circumstances. In what follows, it is not our intention to elaborate explications based on a third truth-value, nor the interpretation based on epistemic explications of the modality of possibility—on the fact that no hindering circumstance is known. Let us just remark that this epistemic interpretation is one of more convincing explanations to the notorious problem of Aristotle's about a sea battle tomorrow.

Truth, and consequently analyticity of the proposition expressed by the sentence (9) would be guaranteed if its component

(9a)   *It will rain tomorrow*

were true regardless of the empirically ascertainable state of affairs. However, it is not such, since there are days followed by days without rain. The sentence (9) is therefore empirical and can be explicated in at least the following two ways:

1) It is true regardless of the time parameter (omnitemporally) in those logically conceivable worlds in which it rains every day. Therefore, it does not refer to logical necessity, but rather scientific necessity—to worlds that form a proper subset of all logically possible worlds. Such worlds are obviously different from the actual world, so this interpretation is hardly feasible;

2) Given a world, it is true in dependence on a time parameter, i.e. in the days followed by the occurrence of all conditions not only sufficient, but also necessary for the 'realization' of rain.

This is also the way we causally explain historical facts, as is the case with the sentence (10): if necessity is taken to mean that the outcome of all circumstances and factors (the Greeks' individual and collective warcraft superiority, their greater determination and motivation, etc.) was the empirical fact of the Greeks' victory over the Persians, then the sentence can be deemed true. This is the approach taken by many historians. This conception of necessity was probably taken by Diodorus Cronus; it is close to strict determinism or fatalism. This is the reason why—provided we assume the Principle of Plenitude stating that whatever is possible will sometime happen—the distinction between the modalities of possibility and necessity is blurred in Diodorus. To understand him right, we should not

associate these modalities with logical ones. Diodorus Cronus' criticism of Aristotle did not concern his explication of logical modalities, but rather that of *empirical* modalities.

Proceeding further, the sentence (11) apparently predicates empirical necessity, accounted for by ideal approximative laws of nature that hold in the worlds compatible with the actual world. Logical modalities are not mentioned here at all.

The sentences (12) and (13) express scientific facts derivable from the laws of nature, while abstracting from specific conditions of the 'realization' of the law. They are thus rather statements about idealized objects (of a negligible size or the shape with a very small drag coefficient etc., and so with negligible air resistance).

Proceeding even further, the sentence (14) speaks of a more complex empirical relationship, reflecting some (major) specific validity restrictions of the ideal law.

It is logically possible that the sentences (11)–(14) need not be true in some worlds, in which the laws of nature are incompatible with those valid in the actual world.

The above examples confirm the hypothesis that we never speak of logical modalities beyond the context of logical scholarly discourse. If we say in the course of ordinary discussion that something is (*really*) *possible* or (*really*) *impossible,* we do not mean anything like *logically conceivable* or *logically inconceivable.* If we say that it is really possible to take a chunk of marble and make of it a life-sized statue of David, and when this is done, we say that it is no more possible to make of it e.g. a life-sized statue of Goliath, then we are not speaking about logical possibility. For it is logically conceivable—logically possible for a chunk of marble fashioned into the statue of David to assume the shape of no smaller Goliath. However, we presume that this is not really possible considering the laws of nature and external circumstances. Such kind of pure logical possibility, the realization of which is prevented by the laws of nature or external circumstances, is usually also called an *abstract possibility.*

4. Logically possible/necessary from the point of view of Transparent Intensional Logic (TIL)

4.1. A logically possible world: modal vs. temporal variability

We cannot reproduce here all the apparatus and origins of TIL, so we shall restrict ourselves to the most necessary requisites for our goals. A set of mutually non-contradicting facts is called a consistent or non-contradicting set. A maximal consistent set of logically

conceivable facts is called a **state of affairs** or *possible world,* taken just modally (i.e., atemporally). The world is not a totality of things, but rather a totality of facts.

Empirical propositions are marked by the feature that the truth-value they codetermine can change in time. The sentence 'It rains' expresses a proposition that determines some truth conditions and it is either true or false, depending on a time point. Taking the time dimension into consideration, a temporal sequence of states of affairs (the modal conception of possible worlds), i.e. a **chronology of states of affairs,** will be called a **possible world,** meaning a **possible history.** Modal variability—the fact that at a given moment things could be different—has often been construed in close relation to temporal variability. Intuitively, however, modal variability is a 'dimension' of alternatives independent of the temporal dimension. The actual distribution of properties (roles and relations) to individual things at a given moment is not the only possible one. Necessity attached to modal variability is different from necessity conceived as omnitemporality: validity in every alternative state differs from validity at every moment.

It is a matter of course[2] that at every moment only one of all possible consistent sets of possible facts is the **set of the actual facts.** The temporal sequence of all such sets of actually valid facts, i.e. the **chronology of the set of actual facts,** is called the **real** (or **actual**) **world.**

4.2. Are logical modalities classes of propositions, or rather properties of propositions?

TIL is a theory of types, based on four atomic types: o—the type of truth-values; ι—the type of individuals; τ—the type of time points/real numbers; and ω—the type of possible worlds. Complex types are sets of partial functions over this base. Specific features of the TIL system are constructions conceived as abstract procedures. Intensions are functions assigning chronologies of a given type to possible worlds, i.e. their type scheme is $\omega \Rightarrow (\tau \Rightarrow \alpha)$, where α is any type. The constructions for intensions are usually of the form

$\lambda w \lambda t\, A,$

where w is a variable ranging over possible worlds, t is a variable ranging over time points and A is a construction that constructs (possibly in dependence on a valuation) an object of type α. Propositions are objects of type $\omega \Rightarrow (\tau \Rightarrow \alpha)$, abbreviated $\alpha_{\tau\omega}$, i.e. objects constructed by constructions of the form $\lambda w \lambda t\, A$, where A is a construction which contains the variables w and t and constructs—in dependence upon w and t—a truth-value (if it is defined).

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[2] Unless we want to assert, like David Lewis (*On the Plurality of Words*, 1986, 73), that alternative possible worlds possess a similar existence with respect to the actual world, only they 'are' in alternative spaces. Such a highly absurd conception would mean that we have our counterparts in each of the alternative worlds.

From the point of view of TIL, logical necessity and logical possibility are classes of propositions. Since propositions are functions from world-time points to truth-values (i.e. objects of type $o_{\tau\omega}$), the expressions $\forall$, $\Box$, $\Diamond$ denote classes of propositions, i.e. objects of type $(oo_{\tau\omega})$. If p ranges over propositions and p_{wt} is short for the application $[[pw]t]$, logical modalities can be defined as follows:

Def. of **logical necessity:** p is logically necessary $\leftrightarrow_{df} (\forall w)(\forall t)p_{wt}$

Logical necessity is the class of propositions that are true in all worlds and time points. This class contains just one proposition—the one which is logically true (i.e. the function that assigns to every possible world a chronology that assigns the truth value 'true' to every time point). Logical possibility is the class of all propositions that are true in at least one world at some time point, in other words those that are neither contradictions nor functions undefined in every world-time:

Def. of **logical possibility:** p is logically possible $\leftrightarrow_{df} (\exists w)(\exists t)\, p_{wt}$

Thus, $\Box$ is modelled as $\lambda p(\forall w)(\forall t)\, p_{wt}$ and $\Diamond$ as $\lambda p(\exists w)(\exists t)\, p_{wt}$. Both modalities are of type $(\omega \Rightarrow (\tau \Rightarrow o)) \Rightarrow o)$, which can be rendered as $(oo_{\tau\omega})$ in TIL notation. This, however, means that both modalities are *extensions,* i.e. classes of propositions, not *properties* of propositions. In other words, which propositions are necessary and which of them are possible is decided 'once for ever' and is independent of the state of affairs (i.e., a possible world and a time point). This is why Pavel Materna says:

'In contrast with modalities in other systems than S5, they cause that any construction of the form $\Box A$, $\Diamond A$ constructs always **T** and **F**, never a proposition. The concept "of necessity" or "of possibility" in those "non-S5" systems are simply other concept than those of *logical* modalities.' [Materna OM].

This, however, shows that the examples of the ordinary use of modalities represented by the sentences (4)–(14) need another explication than that offered by S5.

Note: As regards the sentences of the type

(2)   It is necessarily the case that marble statues are marble,

[Materna–Jespersen] try to show that from the antiessentialist point of view (if an essence is assigned to intensions only), a serious doubt is cast upon their validity.

5. New types of logical modalities and the Principle of Plenitude modified

Distinguishing between modal and temporal variability, additional combination-based types of logical modalities be considered:

1. $\lambda p(\forall w)(\exists t)p_{wt}$—the class of propositions that are satisfiable in each world. They satisfy a modification of the Principle of Plenitude—everything 'possible' (in the sense 'belonging to this class of propositions') will happen in *each* world at *some* (respective) time-point.

2. $\lambda p(\exists w)(\forall t)p_{wt}$—the class of logically accidental, but for some given world eternal propositions.

3. $\lambda p(\exists t)(\forall w)p_{wt}$—the class of proposition that are at a certain time point satisfied in every world. These satisfy another modification of the Principle of Plenitude—everything possible (in the sense 'belonging to this class of propositions') will happen at *some* (common) time-point in *each* of the worlds. The class 3 is a subclass of the class 1.

4. $\lambda p(\forall t)(\exists w)p_{wt}$—the class of eternally satisfiable, but logically accidental propositions. The class 2 is a subclass of the class 4.

Even trying to make a favourable interpretation, one could hardly find examples of propositions from the classes 1 and 3. Propositions from the classes 2 and 4 could be interpreted as ones similar to the laws of nature. The problem is, however, that they are *classes of propositions,* i.e. *logical modalities.* Empirical modalities should apparently be objects of another type.

6. Scientific possibility/necessity

Materna in *Ordinary Modalities* says approximately the following:

Intuitively, empirical modalities (necessity in the manner of the laws of nature or their consequences) should

a) delimit for each possible world a class of propositions, i.e. they should be of type $(\omega \Rightarrow (\omega \Rightarrow (\tau \Rightarrow o)) \Rightarrow o))$—in TIL notation $(oo_{\tau\omega})\omega$;

or

b) delimit for each time point a class of propositions, i.e. they should be of type $(\tau \Rightarrow (\omega \Rightarrow (\tau \Rightarrow o)) \Rightarrow o))$ —in TIL notation $(oo_{\tau\omega})\tau$.

This suggests four types of modalities:

i. $\lambda w\lambda p(\forall t)p_{wt}$, ii. $\lambda w\lambda p(\exists t)p_{wt}$, iii. $\lambda t\lambda p(\forall w)p_{wt}$, iv. $\lambda t\lambda p(\exists w)p_{wt}$. We do not know yet how the two latter types could help explicate modalities in sentences like (4)–(14). The candidate for being an explication of scientific necessity is the type i.

In order to find out whether something is a mathematical truth or not, a mathematician need not study the empirically ascertainable state of affairs, because mathematical truth is independent of how the things stand. On the contrary, scientists—physicists, chemists, biologists, etc.— have to study empirical reality to find out what is a law of nature and what is not. The constructions represented by the formulations of the laws of nature are not direct constructions of the truth-value 'truth' as they are in the case of mathematical or logical 'valid' constructions. Laws of nature, however, differ from 'ordinary' empirical statements of the sort 'It will rain tomorrow' as well.

The formulation **A** of a law of nature should denote an eternal proposition, which in those worlds where the law holds is true at every moment. As such, it delimits a certain class of possible worlds—a proper subclass of all possible worlds. The sentences used for expressing the laws of nature are therefore objects of type $(\omega\Rightarrow o)$, and in this sense they lack temporal variability.

Materna proposes that natural necessity **N** defined as in i. and typed $(\omega\Rightarrow(\omega\Rightarrow(\tau\Rightarrow o))\Rightarrow o))$, or $(oo_{\tau\omega})\omega$ in TIL notation, be called *atemporal property of proposition*.

In the next part of the paper, Materna successfully distinguishes omnitemporal sentences of the sort

(15*)   The Greeks defeated the Persians at Marathon in 490 BC

from the law-like sentences of the revised definition of necessity.

The explication of scientific necessity is, of course, just a starting point for the explication of empirical modalities. If we revert to the Stoics' definitions of modalities, we can see that they obviously describe empirical modalities, defined as some subclasses of logical modalities. If | is to denote the propositional binary connective *is incompatible with* (*is inconsistent with*) and 'Ext' to mean *the external circumstances* in the sense of conjunction of the true empirical propositions including the laws of nature and their consequences, the Stoics' definitions can be reconstructed as follows:

(EP) p is empirically possible $\leftrightarrow_{df} \Diamond p \wedge \neg(p|\text{Ext})$

(nonEP) p is empirically *not* possible $\leftrightarrow_{df} \Diamond\neg p \vee (p|\text{Ext})$

(EN) p is empirically necessary $\leftrightarrow_{df} p \wedge (\neg\Diamond\neg p \vee (\Diamond\neg p \wedge (\neg p|\text{Ext})))$

(nonEN) p is empirically *not* necessary $\leftrightarrow_{df} p \wedge (\Diamond\neg p \wedge \neg(\neg p|\text{Ext})$

The question remains whether they based logical modality just on temporal variability or also on modal variability proper. We would expect that the Stoics considered just temporal variability, as seen with Diodorus Cronus. Contrary to him, however, they refused the Principle of Plenitude (that whatever may happen will happen).

From the point of view of philosophy of science, another important factor should be taken into account: there are many kinds of laws of nature, and the best known laws of physics are formulated as idealizations, while the *modification conditions* are supposed to be negligible, constant or approaching infinity, as e.g. in the sentence (13). If a new type of variability—the *variability of conditions* of idealized laws—is considered, as seen in the sentence (14), a new range of problems opens.

7. Conclusion

In ordinary discourse, the terms 'necessary'/'possible' are usually related to empirical modalities, which are much different from the logical ones. For an adequate account of logical modalities, it is relevant to distinguish between modal and temporal variability. Laws of nature are formulated as idealized atemporal propositions, i.e. they lack temporal variability, but a new variability of conditions emerges. Scientific necessity, explicated as the atemporal property of propositions, is only one model of empirical modalities among many (even though a basic one).

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