Anaphora and notional attitudes[1]

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1. Introduction

When dealing with coherence and cohesion of a text, anaphora and cataphora appear to be important means of retrospective and perspective linking. The scope of an anaphora may be limited by a sentence but it can also extend the sentence to a broader text. Since cataphora differs from anaphora only by the orientation of a reference, and since all the results concerning anaphora can be (after the respective reversion of the orientation) applied to cataphora as well, it is sufficient to deal only with the problem of anaphora. The expression, to which (or to a part of which) an *anaphoric expression* refers, will be called an *antecedent.*

Anaphora, together with an ellipsis (most frequently a syllepsis), makes it possible to "compress" the text - it shortens sentences and/or the text. But, to prevent semantic ambiguities that might be caused by these simplifications, it is necessary that a competent user of a language (a speaker) is able to correctly identify particular elements participating in the anaphoric relationship, and to correctly determine the meaning of a referring word, which in most cases does not pose any additional special demands at the user, he may handle with a normal language intuition. Thus anaphora is sometimes even not considered to be a problem. So in the previous sentence (and in this one) the word *which* is used in an anaphoric way, which does not make the sentence more difficult to understand. This may also be the reason of not introducing anaphoric rules in language guides and textbooks. On the other hand these circumstances may reflect the fact that we deal with a primarily semantic problem, common to many languages, and not with a specific grammar problem of a particular language. Still, it has turned out that some kinds of anaphoric phenomena are not at all trivial and an explication of their semantics may be a hard nut for many logicians/semanticists.

2. Examples and approaches

2. 1. Anaphora as a pronoun coreferring with a singular expression (proper name)

Using the so-called referential theories, in which the relation between an expression and that what is denoted (referred to) is primary and the inference relation secondary, several explications of the anaphoric phenomenon have been provided. A trivial explication is that one based on the coreference. In case of the sentence

(1) John is walking and (he is)[2] talking

the meaning of the anaphoric expression *he* is determined by the meaning of its relevant antecedent. Anaphoric pronoun referring to a singular expression refers to the same thing as

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[2] In English (as well as in Slovak language) pronouns are usually omitted in such sentences. Since in this way the anaphoric phenomenon would not be represented in its full scope on the language level, we will use these referring expressions in the sentence structure, which will enable us to abstract from specific features of English (Slovak language), and the anaphoric phenomenon can be analysed as a general semantic problem.

the previous proper name: pronoun *he* refers to the same individual as the name John in the first component of the conjunction. Hence sentence (1) should be semantically equivalent to the following sentence:

(1*) John is walking and John is talking

2. 2. Anaphora as a **variable bound by an existential** quantifier

Analysis of the following sentences

(2) Somebody is walking and somebody is talking
(3) Somebody is walking and (he is) talking

shows that the pronoun *he* in the sentence (3) does not refer to the same individual as the second occurrence of the pronoun *somebody* in the sentence (2), which is proved by their transcription to the language of 1st order predicate logic:

(2*) $(\exists x)(W(x)) \wedge (\exists y)(T(y))$
(3*) $(\exists x)(W(x) \wedge T(x))$.

Here the anaphoric use of the pronoun *he* can be explained by means of a variable in the scope of the same existential quantifier that bounds the variable expressed by the pronoun somebody.

2. 3. Anaphora as a **variable bound by a general** quantifier

In the sentence

(4) If somebody is walking then he is moving

the meaning of the pronoun *he* can be explicated by means of the general quantification - the traditional analysis of sentences like this shows that we deal with a general statement:

(4*) $(\forall x)(W(x) \rightarrow M(x))$,

and the anaphoric expression plays the role of a variable bound by a general quantifier.

But what about indefinite descriptions like *(some)F,* where “F” is a name of a property, and this description is referred to by an anaphoric expression? One of the well-known ”donkey” sentences

(5) Each man who bought *some* donkey beat *him*

would be, in accordance with Geach [1962, 126ff], captured in the 1st order predicate logic in this way:

(5*) $(\forall x)\, (\forall y)((M(x) \wedge D(y) \wedge \mathrm{Buy}\,(x, y)) \rightarrow \mathrm{Beat}\,(x, y))$,

where property M is the property of *being a man,* D *being a donkey,* Buy the relation of *buying...,* Beat the relation of *beating...*

This would mean that an indefinite description does not induce existential presupposition and together with the anaphoric expression create a couple expressing the general quantification.

2. 4. Anaphoric pronoun as a **variable bound by a restricted quantifier**

Another analysis of sentences like (4), (5) together with the intention to explain all the known cases of anaphora has been introduced by Russell's followers who modified and/or generalised Russell's theory of descriptions. For instance, *determiners* (quantitative pronouns) like “some“, “all“, “most“, “no“, “none“, (in English also articles “the“, “a“, “an“) are understood as elements of a unified syntactic category. Though sentences, in which the first two are used, can be analysed by formulae of the form:

(6) (Qx)(F(x)©G(x)),

where Q is a quantifier (determiner) and © is a propositional connective, in case of the pronoun “most“ this form cannot be used in a predicate logic. Thus Geach proposed a unified and general solution that has been further used and developed by, e.g., neorussellian Neale [Neale 1990, 40ff]: these determiners are understood as means that can be combined with two simple or complex formulae (or predicates) and form a formula. A determiner can be combined with a formula (or predicate) and form the so-called *restricted quantifier*. For instance, “most“ combined with “man“ creates a restricted quantifier “most people“, which is written as:

(a) [most *x*: man *x*]

This quantifier can be combined with another formula “to be mortal“, which creates a *formula* with bound variable:

(b) [most *x*: man *x*] (mortal *x*).

In sentences like:

(7) Some boys suppose that they are immortal,

the word “some“ fulfils, according to neorussellians, the role of a quantifier and the pronoun “they“, which refers to it, is actually a variable bound by it:

(c) [some *x*: boy *x*] (x supposes (immortal *x*)).

Supposing that definite descriptions are quantifiers, a variant of the sentence (7) with the definite article would be:

(8) The boy thinks that he is immortal

and its analysis:

(8*) [the *x*: boy *x*] (x thinks (immortal *x*)).

The anaphoric pronoun would again refer to quantifier and fulfil the role of a variable bound by it. An analysis of (4) would be:

(4**) [every *x*: walking *x*] (moving *x*).

There is, however, another reason for refusing analysis (5*) of sentence (5). According to Russell[3] indefinite descriptions are existentially quantified expressions - they demand satisfaction of the existential presupposition, which is not ensured by the formula (5*). This demand might be met in a more proper way by a transcription:

(5**) $(\forall x)((\mathrm{M}(x) \wedge (\exists y)(\mathrm{D}(y) \wedge \mathrm{Buy}\ (x, y)) \rightarrow \mathrm{Beat}\ (x, y))$,

in which, however, the last occurrence of variable *y* is out of the scope of the existential quantifier that should bind this variable. If we insist on existential understanding of the expression *(some) donkey* in the sentence (5), then it seems that we are not able to express it by an adequate formula.

Neale proposed a solution, according to which on the one hand the existential character of indefinite descriptions is preserved (Russell's demand) and on the other hand the anaphoric pronoun expresses general quantification (Geach's truth conditions):

(5_N) [every *x*: man *x* $\wedge$ [some *y*: donkey *y*] (*x* beat *y*)][4].

A problem: referring to singular expressions in case of attitudes

Explication of the anaphora phenomenon based on the idea of determiners as restricted quantifiers meets however some problems even when analysing referring to a typical singular expression, like, e.g., a proper name or demonstrative when dealing with attitudes. In sentences:

(9) Daniel loves **his** mother
(10) Joan thinks that Daniel loves **his** mother

we could manage with the coreference of anaphoric pronouns and proper names. But if we explicated in this way also a sentence:

(11) Joan thinks that he is not Daniel,

then we would deduce that Joan thinks a contradiction in case that the individual about whom Joan thinks that actually is Daniel and she does not know it:

(12) Joan thinks that Daniel is not Daniel.

We could avoid this undesirable result in several ways. *First,* if we distinguished the **sense of a proper name** from its **referent.** *Second,* if we distinguished **referential using of a proper name** from the so-called **attributive** one**.** *Third,* if we essentially changed their semantics and considered (in accordance with Richard Montague's opinion) **demonstratives** and **grammar**

[3] [Russell 1905].
[4] [Neale1990, 236].

proper names to be **quantifiers** rather than singular expressions. Scott Soames characterizes the third approach as follows:

"Anaphoric pronouns c-commanded by such antecedentc would not inherit the sense or reference of their antecedents, but rather would function as bound variables." [Soames 1994, 256]

Hence the above sentences can be represented in the following way:

(9*) [Daniel *x*](*x* loves *x*'s mother)
(10*) [Daniel *x*](Joan thinks that *x* loves *x*'s mother)
(11*) [some *x*: man *x*](Joan thinks that *x* is not Daniel)

But Soames avoided a more detailed semantic analysis of the propositional attitude - he only paraphrases it, and his solution blocks only one undesirable consequence. Sentence (9) expresses a proposition that according to Soames is a structured complex composed of a property of loving own mother and a higher-order property expressed by a quantifier. In case of sentence (9) this higher-order property is that one, which is true exactly for those properties that are true for Daniel. Hence the proposition expressed by sentence (9) is as follows:

(9b*) ⟨property *to be Daniel's property*, property of *loving own mother*⟩.

But this is in a way deflection from Montague's conception of a proposition as a set of possible worlds, and on the other hand the explication of a proposition as a complex by means of the set notion is peculiar: complex should be something that has a structure, parts, but sets, including sets of ordered tuples, are in principle simple, flat, unstructured entities.[5] That means that even this third proposal is not a way out of the "blind alley".

2. 5. Anaphoric pronoun as a **variable bound by the operator of abstraction**

Similar results in explaining the anaphora phenomenon, as the above three proposals of proper names explication achieve, can be achieved also in such a way that keeps conceiving grammar proper names as singular expressions but explicates the anaphoric pronoun as a variable bound by the operator of **abstraction** introduced by an anaphoric relationship:

"...anaphoric pronouns with c-commanding singular term antecedents are variables bound by an abstraction operator introduced by the anaphoric relation itself." [Soames 1994, 257]

Analysis of sentence (9) is:

(9**) λ*x*(*x* loves *x*'s mother) Daniel.

In this case a proposition should be the complex:

(9b**) ⟨Daniel, the property of *loving own mother*⟩.

[5] See the discussion [Cmorej – Tichý 1998].

However, it is again not clear which components could be parts of this complex, because we actually again deal with a set that is in principle without any parts (though it has elements, members of the set, but these are not its components).

2. 6. Anaphoric pronoun as a **hidden description,** i.e. **a restricted quantifier**

In case of sentences like:

(13) Daniel thinks that every man who has an appointment with an (**some) intelligent woman** loves **her**

it seems that[6] the indefinite description "(some) intelligent woman" as a restricted quantifier is an antecedent of the pronoun ("her"), but it cannot be said that the description c-commanded the pronoun. Hence the description should be conceived as a free variable that is not bound by its antecedent, or as a hidden description, i.e., according to neorussellians, a restricted quantifier.

Sentence (13) denotes the same proposition as the following one:

(13*) Daniel thinks that every man who has an appointment with an (some) intelligent woman loves **(that) intelligent woman** ([or] **all the intelligent women**) with whom he has the appointment(s).

Neale [1990, 230 ff] moreover explains that most of the pronouns, which are anaphoric to indefinite descriptions, are numerically neutral, which leaves a space to instantiation depending on the speech situation – context, linguistic factors, etc.

3. Open problems

Soames [1994] examined all the known approaches to solving anaphora in connection with (propositional) attitudes and he has showed that each of the five examined explications (our (1) and (3) – (6)) fails in some cases.

We should take into account the fact that there are also other approaches to the problem of anaphora. In natural language semantics based on the so-called *dynamic logic* anaphora is explicated as a **selection function** in the semantics of *context -change potentials*. In this respect, one of the most sophisticated standpoints has been presented by Jaroslav Peregrin [1999, 2000]. Since this is a theory that is based primarily on the notion of inference (that is considered to be a basic one), and our approach considers it to be a derived semantic notion (i.e. our theory is a referential one), we will not deal with the semantics of context-change potentials any more, since the two approaches are hardly comparable.

4. Anaphora from the viewpoint of Transparent Intensional Logic (TIL)[7]

4. 1. TIL potential to solve the problem of anaphora

1. TIL is a theory that takes the semantic relation of denoting as a primary one, and the relation of logical consequence is defined only on the basis of denoting. In this sense it is a **"referential" semantics**, but not in the sense of common understanding of the word "reference": the meaning of an expression is generally not empirically "anchored" (this

[6] See [Evans 1977], [Davies 1981], [Neale 1990], [Soames 1994].
[7] The author of TIL is P. Tichý – see especially [Tichý 1988].

type of reference is correctly criticised for instance by Peregrin [2000]). An expression talks about an object that is generally not located in space and time.

2. In the TIL semantics **the meaning of an expression** is not reducible to a kind of a function over a base (consisting of individuals, possible worlds, etc.), but it is, in principle, **structured** entity.
3. TIL is based on the **theory of types**. Hence it makes it possible to distinguish (infinitely) many types of referring links: to (extensional entities) like individuals, sets of individuals, relations between individuals, etc., but also to intensional entities (functions from possible worlds …) like individual offices, propositions, properties, etc., and finally it enables us to distinguish also infinitely many relations to structured entities (constructions). This is a great benefit of TIL, which enables us to semantically analyse the so-called propositional attitudes in an adequate way, which is an unsolvable hard nut for the most other (otherwise powerful) semantic theories.
4. Since on the one hand TIL is inspired by λ-calculus and all the operators are definable by means of abstraction, and on the other hand definite descriptions are considered to be semantically self-contained names of individual concepts, the counter-intuitive neorussellian explication of definite descriptions as quantifiers is in TIL rejected.
5. Due to the fact that in case of names of functions TIL enables us to analyse a fine-grained distinction between the ***de re*** and ***de dicto*** **supposition**, the cases when the existential proposition must / must not be satisfied are convincingly explicated and defined [Duží 2000, 2001]. The *de re* / *de dicto* supposition is sometimes indicated by the scope of quantifiers, but the TIL semantics provides a deep explication of reasons why and which scope is justified.
6. TIL makes it possible to explicitly separate the two "worlds" – reporter's perspective ("speaker's world") and believer's perspective (the "world" of the individual to whom the attitude is ascribed). Thus the principle of the dominancy of the *de dicto* supposition is modified [Duží 2000, 375 ff], and the notion of information independence is exactly explicated in the connection with the *de re / de dicto* distinction [Duží 2001, 242 ff].
7. Reflexive "anaphora" does not have to be considered as an actual anaphora, which is in a good accordance with Tichý's conception.

We will concentrate mainly on the semantics of anaphoric expressions in the context of propositional and notional attitudes.

5. TIL semantics

Since the terminology in this area is generally not used unambiguously, we will keep the terminology and the results of developing original Tichý's ideas, as they were presented, e.g., in Materna [1998], [2000]. Just a brief summary: an expression expresses its *sense (=meaning)* that identifies (non-fregean) *denotation*. Hence an expression denotes (talks about) its denotation via its meaning-sense.[8] The sense of an expression is in principle a structured "procedure" – a closed construction (or the concept specified by it). A construction identifies (constructs) the denoted object that is (in case of a "successful" constructing, i.e. construction not being improper) an intension or extension (a first-order, set-theoretical object), or even a higher-order object (involving a construction). Empirical expressions always denote an intension, and in this case we speak also about a referent or *reference* of an expression, which is the value of the denoted intension in a given world/time. The relation between the first-order object (intension) and that what is in most semantic theories

[8] This is the most significant divergence from the original Tichý's proposal, according to which an expression denotes its referent (construction). [Tichý 1988, 224].

considered to be a reference of an expression (for instance an individual in space and time, a set of individuals, etc.) does not have a semantic character; it is influenced by an empirical factor – state of affairs, and thus it is not directly a subject of a semantic-theory investigation. Hence a co-reference of expressions is from the TIL view point a contingent, empirical matter. Expressions can be equivalent when they denote one and the same object, but do not have (even in this case) to have the same sense, i.e. do not have to be synonymous.

Definition 1 **(Types over a base** in the simple theory of types):
(An objectual) base is a collection of mutually disjoint nonempty sets.

i) Every member of the base is a *type over base.*

ii) Let $\alpha, \beta_1,..., \beta_m$ are *types over base,* then $(\alpha\ \beta_1... \beta_m)$, i.e. the set of all (partial) functions with an argument (a tuple) $\langle b_1... b_m\rangle$, where b_i $(1 \le i \le m)$ is a member of the type β_i, and at most one value of type α, is a *type over base*.

iii) Nothing is a *type over base* unless it so follows from i) - ii).

An object (that is a member) of a type α will be denoted an α-object.

Definition 2 (**Constructions**):

i) *Variables* are **constructions.** Variables and constructions involving variables construct objects depending on valuation, they *v*-construct.

ii) If X is an entity whatsoever, even a construction, then ^0X is a **construction** called *trivialisation.* Trivialisation ^0X constructs X without any change.

iii) If X_0 is a construction that *v*-constructs a function (mapping) F, i.e. an $(\alpha\beta_1...\beta_n)$-object, and $X_1,..., X_n$ are constructions that *v*-construct β_1-, ...,β_n-objects $b_1,..., b_n$, respectively, then $[X_0\ X_1\ ...\ X_n]$ is a **construction** called *composition.* If F is defined on the argument $\langle b_1,..., b_n\rangle$, then composition $[X_0\ \ X_2...\ X_n]$ *v*-constructs the value of F on $\langle b_1,..., b_n\rangle$; otherwise it does not construct anything, it is *v-improper.*

iv) Let $x_1,... , x_n$ be pair-wise distinct variables that range over types $\beta_1, ..., \beta_n$, and let X be a **construction** that *v*-constructs an α-object for some type α. Then $[\lambda x_1...x_n X]$ is a **construction** called *closure* (abstraction). It *v*-constructs the following **function F** (of the type $(\alpha\ \beta_1\ldots\beta_n)$)**:** Let *v´* be a valuation that differs from *v* only by assigning objects $b_1,..., b_n$, (of the respective types) to variables $x_1,\ldots,x_n$, respectively. Then the value of the function F on the argument $\langle b_1,..., b_n\rangle$ is the object *v´*-constructed by X. If X is *v´*-improper, then F is *undefined* on the given argument.

v) Nothing is **construction** unless it so follows from i) - iv).

Notes:

1. The simplest constructions are variables; they are the only atomic constructions, all the other constructions have (proper) parts. Variables are open constructions that construct objects dependently on valuation (they *v*-construct).
2. Obviously, the simplest composed construction is a trivialisation. It consists in grasping an object and its "delivering" without any change. If X is an entity, then ^0X simply constructs the entity X.
3. A composition corresponds to the traditional operation of *application* (of a function to an argument). Composition may be (*v*-)improper in three cases: *First,* the component X_0 constructs a function F and components $X_1... X_n$ construct $\langle b_1,..., b_n\rangle$, but F is not defined on this argument. *Second,* objects $b_1,..., b_n$ are not of proper types to create an argument of F. *Third,* some of the components $X_1... X_n$ fail(s) to construct an object b_i (is *v*-improper).

4. Closure (λ-abstraction) enables us to construct a function, and thus also to analyse talking about the whole function (to "mention" it), not only talking about a particular value on a given argument (to "use" the function).

Quantifiers − general $\forall$ and existential $\exists$ − are functional objects of type $(o(o\alpha))$. We will write $(\forall x)A$, $(\exists x)A$ instead of $[{}^0\forall_\alpha\, \lambda_\alpha xA]$, $[{}^0\exists_\alpha\, \lambda xA]$, respectively. Singulariser I_α is an object of type $(\alpha(o\alpha))$, and instead of $[{}^0I_\alpha\, \lambda x\, A]$ we will use I*x*A (*the only x such that A).* We will also use a standard infix notation without trivialisation in case of using truth value functions ($\wedge$, $\vee$, …), but we have to keep in mind that these are only abbreviations that hide the self-contained meaning of the respective "logical symbols".

Definition 3 (**Ramified theory of types**)
Let B be an objectual base, i.e. a collection of mutually disjoint non-empty sets.

1. Types of order 1

(T_1i) Every member of the base B is a *type of order 1 over B.*

(T_1ii) Let α, β_1,..., β_m (0< m) be *types of order 1 over B.* Then ($\alpha\ \beta_1$... β_m), i.e. the set of all *m*-ary (total and partial) functions from β_1 x... x β_m to α, is a *type of order 1 over B.*

(T_1iii) Nothing is a *type over 1 over base B* unless it so follows from (T_1i) and (T_1ii).

2. Constructions of order *n*

(K_ni) Let α be a *type of order n over* B. If ξ is a variable that ranges over α, then ξ is a *construction of order n over B.*

($K_n$ii) If X is a member of a type of order *n*, then ${}^0X$ is a *construction of order n over B.*

($K_n$iii) If X_0, X_1,..., X_m are *constructions of order n over B,* then $[X_0\ X_1...X_m]$ is a *construction of order n over B.*

($K_n$iv) If distinct variables x_1, ..., x_m, as well as X, are *constructions of order n over B,* then $[\lambda x_1...x_m X]$ is a *construction of order n over B.*

Let $*_n$ be a set of all constructions of order n over B. Types of order *n*+1 over B are defined as follow:

3. Types of order *n*+1

(T_{n+1}i) $*_n$ and all the types of order *n* are *types of order n+1 over* B.

(T_{n+1}ii) If α, β_1, ..., β_m are *types of order n+1,* then the set ($\alpha\beta_1...\beta_m$) of all *m*-ary (total and partial) functions from β_1 x... x β_m to α is also a *type of order n+1 over B.*

(T_{n+1}iii) Nothing is a *type of order n+1 over B* unless it so follows from (T_{n+1}i) a (T_{n+1}ii).

An object O of a type α will be called an α-object, written also O/α.

An objectual base is a special kind of base, over which (an infinite) hierarchy of functions and constructions can be built up, and our conceptual scheme can be adequately modelled within this system. This base consists of sets of objects of four basic categories $\{\iota,o,\omega,\tau\}$, where ι? is a type (set) of **individuals**. The objectual base together with the interpretation of other elements constitutes an *epistemic frame.* Interpretation of its other elements is as follows: o is the type (set) of **truth-values** {T, F}; ω is the type (set) of **possible worlds,** and τ is the type (set) of **time points,** or **real numbers** (playing the role of their surrogates). The collection of pre-theoretically given (basic) features (traits), using which all the other notions are defined, constitutes the *intensional base* of the given system.[9]

Empirical expressions denote *intensions*, i.e. functions from possible worlds and time points (world/times) to a type α. Hence α-intensions are functions of type $((\alpha\tau)\omega)$, which will

[9] For details, see [Tichý 1988, 201ff].

be abbreviated by $\alpha_{\tau\omega}$. We will standardly use variable *w* as ranging over ω, and variable *t* as ranging over τ. If X is a construction that constructs an intension of type $\alpha_{\tau\omega}$, then instead of $[[Xw]t]$ we will write X_{wt}.
Examples of intensions: Individual concepts are objects of type $\iota_{\tau\omega}$, properties of individuals are objects of type $(o\iota)_{\tau\omega}$, binary relations-in-intensions between individuals are objects of type $(o\iota\iota)_{\tau\omega}$, propositions are objects of type $o_{\tau\omega}$.

6. Propositional attitudes

An analysis of some "extreme situations", typical representatives of which are indirect ("oblique") contexts, is a stumbling block of semantic theories. It was already Gottlob Frege who realised that his standard semantic scheme is not valid in indirect context: in clauses of a certain type their original referents – truth-values do not "come into play", but their senses matter.[10] This fact brought him to the revision of the semantic scheme, but only for these indirect contexts. On the other hand Frege[11] came up with a genial idea: Every subexpression of a statement can be conceived as a name of a function or a name of an argument of a function, which has lead to the application of functional approach to the analysis of a language. However, in case of a statement itself he supposed that the statement does not have a functional character any more; it is just a name of a truth-value. This has decreased the explicative power of such semantics.

Introducing the so-called *state descriptions* enabled Rudolf Carnap[12] to overcome this restriction: since state descriptions are some language counterparts of possible worlds, he actually conceived propositions (as *intension* expressed by the statements) as functions from possible worlds. This made it possible to systematically distinguish between *intensions* and *extension.*

If we want to keep the meaning of an expression unchanged, we can substitute for any expression only an expression that denotes the same intension. This should hold also for the so-called hyperintensional contexts that used to be traditionally called *intentional.* They are the contexts, in which attitudes to propositions are expressed (*propositional attitudes*). Substituting in this way into the sentence

(14) Richard knows that London is greater than Oxford,

we obtain a sentence that should have the same meaning:

(15) Richard knows that Oxford is smaller than London.

Carnap realised that substituting L-equivalent expressions with the same intension for some clauses does not keep up their semantic contribution in "belief contexts", and formulated a stronger criterion of the so-called intensional isomorphism between substituted expressions. Benson Mates [1950] was perhaps the first one who pointed out some difficulties connected with this demand, and Alonzo Church [1954] demonstrated its principal incorrectness.

A fundamental revision of a semantic scheme, which makes it possible to adequately explicate the "behaviour" of expressions even in hyperintensional contexts, has been presented by Pavel Tichý. The sense of an expression is conceived as a construction, and in these contexts we are dealing with attitudes to constructions that construct propositions; hence they are attitudes to *propositional constructions.* Thus what has been called "propositional attitudes" are just attitudes to propositional constructions. Besides these

[10] [Frege 1962].
[11] [Frege 1964, VII]
[12] [Carnap 1947].

attitudes we also often deal *notional attitudes*. In general, they are relations-in-intension between an *individual* and *the structured meaning* of the embedded expression, which are expressed by verbs like *to believe, to think, to know, to doubt, to seek,* etc.

Sentences like

(16) X *supposes* (*is convinced, believes, thinks, ...*), that 5 + 7 = 12,

can be analysed as follows: $D/(o\iota*_1)_{\tau\omega}$ is a “propositional” attitude, or more precisely an attitude to the construction of a proposition, 5, 7, 12 are τ-objects, + is an object of type (τττ) and =/(οττ):

$$[\lambda w \lambda t\, [{}^0D_{wt}\ {}^0X\ {}^0[{}^0{=}[{}^0{+}\ {}^05\ {}^07]{}^012]]].$$

The trivialisation of the construction $[{}^0{=}[{}^0{+}\ {}^05\ {}^07]{}^012]$ is here indispensable, for otherwise D would be an attitude to the truth-value True (or, as the case may be, to the proposition *true* (taking the value True in every world/time)). But D is an attitude to the construction of this truth-value, i.e. to a construction of order 1.

Now it is obvious why the following argument is not valid:

(U) Jan knows that Bratislava has 500 000 inhabitants
$\underline{500\,000 = 2^5 \times 5^6}$
$\therefore$Jan knows that Bratislava has $2^5 \times 5^6$ inhabitants

This argument is of the “form”:

$$[{}^0D_{wt}\ {}^0X\ {}^0C_1]$$
$$[{}^0{=}\ \ C_1\ C_2]$$
$$\therefore [{}^0D_{wt}\ {}^0X\ {}^0C_2],$$

where D is a propositional attitude and C_1, C_2 are constructions. This argument scheme is generally not valid, because the second premise states only an equivalence of constructions (the identity of the constructed objects), and not the identity of the constructions themselves. The argument scheme would be valid if constructions C_1 and C_2 not only constructed the same object but were also identical, i.e. the second premise would have to be[13]:

$$[{}^0{=}\ \ {}^0C_1\ {}^0C_2].$$

On the one hand it means that if we have an attitude to a construction expressed by a sentence *p*, then it does not mean that we have this attitude also to sentences that express logical consequences of the sentence *p* (they logically follow from *p*). Hence if we know that 5 + 7 = 12, then we do not have to know all the mathematical truths; or, if we know, e.g., the axioms of arithmetic, we do not have to know all the truths of arithmetic. Otherwise we would have to suppose that the language user is a logical and mathematical genius, that if he knows any mathematical truth then he knows also all its logical consequences. This would not be in accordance with our intuitions and with the principle of *Non-omni*.

On the other hand, however, the demand of the identity of constructions seems to be very restrictive. Identical constructions have to construct the same, but not only that; they must also be „built up“ from the very same constituents, subconstructions.

We might eliminate this restrictiveness by conceiving attitudes to *empirical* embedded clauses as being attitudes to propositions denoted by these clauses (to states-of-affairs) instead of to constructions of these propositions. The sentence (15) would thus follow from the sentence (14). This would, however, mean that only two non-realistic (idealised) types of a

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[13] For details, see [Materna 1998], [Materna – Štěpán 2000, p. 76]. The latter contains, however, an error (misprint) concerning just this argument. (Translator’s note.)

language user are considered: either a logical and mathematical idiot (the former case), or a perfect language user who is a logical and mathematical genius (the latter case), and in this restricted sense omniscient [Duží 2001, 240 ff]. Thus an explicit propositional attitude would be distinguished from an implicit one [Duží – Materna 2000]. Restrictive consequences of the constructional approach are reduced: in case of attitudes to empirical clauses we take into account also the attitude to the states-of-affairs, not only to their constructions. The relation of an implicit propositional attitude is thus closed under the relation of logical consequence (the agent is in a way „analytically" omniscient - if he, for instance, knows that *p,* then he also knows all the constructions that identify *p,* though he does not have to know it „explicitly").

7. Notional attitudes

We may have an attitude not only to the object specified in a sentence by another (sub-)sentence, but also to the object specified by a non-sentential expression, a notion. A typical notional attitude is expressed by the expression *thinking of.* Consider the sentence

(17) John is thinking of Peter,

which can be analysed as follows (T(hinking)/$(o\iota\iota)_{\tau\omega}$):

(17*) $[\lambda w\lambda t\, [{}^0T_{wt}\ {}^0J\ {}^0P]]$.

In this case the special character of intentional verbs is not demonstrated. But consider the sentence

(18) John is thinking of Daniel's best friend.

<u>Analysis</u>: Best friend is a function that (dependently on world/times) associates an individual with another individual (its best friend), i.e. BF(riend)/$(\iota\iota)_{\tau\omega}$.
Now there are three possibilities of assigning type to the object T denoted by 'thinking of'. John can be thinking of:
a) an individual who fulfils the role of Daniel's best friend, T/$(o\iota\iota)_{\tau\omega}$, and we get the analysis:
(18a) $[\lambda w\lambda t\, [{}^0T_{wt}\ {}^0J\ [\lambda w\lambda t\, [\,{}^0BF_{wt}\ {}^0D]]_{wt}]]$;
Note that "Daniel's best friend" (the construction $[\lambda w\lambda t\, [\,{}^0BF_{wt}\ {}^0D]]$ of this office) occurs *de re* (intensional decent to reporter's perspective – "application to" reporter's *w,t*) and the sentence has the existential presupposition (Daniel's best friend has to exist so that the sentence might be true or false).
b) the role ("office") of Daniel's best friend, T/$(o\iota\iota_{\tau\omega})_{\tau\omega}$, and we get:
(18b) $[\lambda w\lambda t\, [{}^0T_{wt}\ {}^0J\ [\lambda w\lambda t\, [\,{}^0BF_{wt}\ {}^0D]]]]$;
Now "Daniel's best friend" (the construction $[\lambda w\lambda t\, [\,{}^0BF_{wt}\ {}^0D]]$ of this office) occurs *de dicto* and the sentence does not have the existential presupposition (Daniel's best friend might not exist, yet John might be thinking of this office, he might, e.g., wish that he did exist, that Daniel had friends, …).
c) the notion (concept) of Daniel's best friend, T/$(o\iota*_1)_{\tau\omega}$:
(18c) $[\lambda w\lambda t\, [{}^0T_{wt}\ {}^0J\ {}^0[\lambda w\lambda t\, [\,{}^0BF_{wt}\ {}^0D]]]]$.

All the three cases are plausible analyses. The sentence (18) is ambiguous.

Case a) might seem to be plausible only in such a situation when John knows who Daniel's best friend is. It is not so. Of course, John has to have a relation to the individual who plays the role of Daniel's best friend, let it be Charles, but Charles may be "submitted" to

John in many other ways without John's knowing anything about the fact who Daniel's best friend is. Still, the reporter may truly report on the situation using (18). Of course, the sentence does not say anything about Charles.

Case b) may seem not to be intuitively possible, if we tacitly assume the a) reading of the sentence. But it is quite comprehensive, and not only in case of non-existence of Daniel's best friend. Imagine, e.g., that Daniel has met with a serious accident and John worries whether his best friend isn't shocked, without having a slightest idea who Daniel's best friend is. Then the analysis (18b) is correct – John is actually thinking just of the individual office not of its value in a world/time.

Case c) seems to be plausible only in special contexts, but obviously it is possible to think of a logical / mathematical entity, and then *thinking* is related to a construction. If, e.g., John is a logician and he is thinking of the concept of Daniel's best friend, whether it identifies an individual office, etc. In such a case an unambiguous reformulation would be, of course, desirable.

The ambiguity of the sentence is caused by the phenomenon of *type polymorphism*[14]. An expression is type polymorphic if it denotes functions that can have arguments of different types (in case of extensions), or intensions that differ by the type of arguments of functions that are values of these intensions in world/time. For instance, identity is type polymorphic, it denotes relations of type $(o\alpha\alpha)$ for any type α. Similarly quantifiers are of type $(o(o\alpha))$ for any type α, etc.

The expression *thinking of* is type polymorphic as well. Its reference (value of the denoted intension) can be a function of type $(o\iota)$, cases (17) and (18a), or a function of type $(o\iota_{\tau\omega})$, case (18b), or even function of type $(o{*_1})$, case (18c), and many others.

Similar ambiguity (type polymorphism) is connected with the expression *to seek.*[15] If we know the identity of the object we are looking for, but do not know, e.g., the place where the object is located, then in the sentence

(19) Pankrac is seeking Bonifac

we do not speak about seeking the individual Bonifac, but about looking for his location in a given time. Hence this kind of seeking is a relation of an individual to the location where another individual is situated in a given time. Paraphrasing (19) as 'Pankrac wants to locate Bonifac' or, more precisely

(19′) Pankrac is seeking (the location of) Bonifac

we get the type-theoretical analysis as follows: P(ankrac) / ι, B(onifac) / ι, L(ocation of) / $((o\tau\tau\tau)\iota)_{\tau\omega}$ is an empirical function that associates (dependently on world/times) each individual with a set of space-point coordinates; denoting type $(o\tau\tau\tau)$ as μ, we have L/$(\mu\iota)_{\tau\omega}$, and LX (location of an individual X) /$\mu_{\tau\omega}$ – a μ-office; S(eeking) is a relation-in-intension of an individual to to this μ-office, S / $(o\iota\mu_{\tau\omega})_{\tau\omega}$. Hence the component "the location of Bonifac" (the constituent – construction of this office $[\lambda w\lambda t\ [^0L_{wt}\ ^0B]]$) occurs *de dicto* (seeking is a relation to the office). If its supposition were *de re*, then it would mean that Pankrac knows that place – it is given to him in some other way, but then he could not look for it.

(19*) $[\lambda w\lambda t\ [^0S_{wt}\ ^0P\ [\lambda w\lambda t\ [^0L_{wt}\ ^0B]]]]$.

[14] See [Duží 1993]

[15] Semantic analysis of *seeking* and *finding* has been presented in [Jespersen 1998], [Jespersen 1999], [Duží 2000].

The existence of the value of the office *the location of the object (Bonifac)* does not matter here, though in case of Bonifac its space-time location has to exist. But we may wish to locate also such an object, about which we are convinced that it does exist, but actually it does not have to. This may happen in such cases when dealing with an entity determined "via" an individual role. Consider the sentence

(20) Walter Raleigh looked for Eldorado,

which can be paraphrased as

(20´) Walter Raleigh looked for (sought) the location of Eldorado.

Now, we may deal with looking for the location of the value of an individual office (*the only land of glory and profusion, flowing with gold that is situated in South America)*[16], or of a value of a property *(a land of glory and profusion flowing with gold that is situated in South America)*. Though Walter Raleigh assumed that something such exists (according to a legend), i.e. that the individual concept or the property is instantiated, but since actually nothing such exists, he could not find the *location* of Eldorado. The *first presupposition* of the existence of an occupant of the office (property) Eldorado has not been satisfied, and thus even the *second presupposition* of the existence of the *location* of Eldorado could not be. Similarly from seeking the (location of) "St. Graal", "water of life", etc., the existence of these does not follow. Sentence (20) is reasonable and (in a given world/time) has a truth-value, if Walter Raleigh really looked for Eldorado, independently of the fact whether the individual office (property) is instantiated (the first presupposition of finding is satisfied). This is given by the fact that the relation of seeking is a relation to an office not to its holder. This has been noticed already by Church, when he claimed that in the sentence "Ponce de Leon searched for the fountain of youth" we deal with a relation to an individual concept.[17] When we assume that the expression "Eldorádo" is an abbreviation of the individual definite description, then the type-theoretical analysis of (20') is as follows:
WR(aleigh) / ι, L(ocation of) / $(\mu\iota)_{\tau\omega}$, S(eeking) / $(o\iota\mu_{\tau\omega})_{\tau\omega}$ a E(ldorado) / $\iota_{\tau\omega}$. The sense of the sentence is:

(20*) $[\lambda w\lambda t\ [{}^0S_{wt}\ {}^0WR\ [\lambda w\lambda t\ [{}^0L_{wt}\ {}^0E_{\ wt}]]]]$.

Again, the subject of seeking $[\lambda w\lambda t\ [{}^0L_{wt}\ {}^0E_{\ wt}]]$ occurs in the *de dicto* supposition and therefore even its constituent ${}^0E$ occurs *de dicto*, though it is intensionally descended (the *wt* "index"), but not to the speaker's perspective. Hence Eldorado does not have to exist. Summary: seeking in the context of the above sentences is a relation(-in-intension) to the office *the location of an object.* The object can be given directly or "via" an office. Hence the principle of existential presupposition, which holds generally in the *de re* cases, does not hold in *de dicto* cases (see Duží [2000]).[18]
Similarly, analysing the sentence

(21) Schliemann sought Troja

[16] Eldorado – from Spanish *the golden (man)*; due to a legend it obtained a metaphorical meaning.
[17] [Church, 1951, note 14].
[18] Similarly the principle of intersubstitutivity of co-referential expressions holds only in *de re* cases.

we get: Sch(liemann) / ι, L(ocation of) / $(\mu\iota)_{\tau\omega}$, S(eek) / $(o\iota\mu_{\tau\omega})_{\tau\omega}$ and T(roja) / $\iota_{\tau\omega}$[19]

(21*) $[\lambda w \lambda t\, [^0S_{wt}\ ^0\mathrm{Sch}\ [\lambda w \lambda t\, [^0L_{wt}\ ^0T_{wt}]]]]$,

where the subject of seeking is again not a location of the value of an individual office, but the whole μ-office. This in a good accordance with our intuition, for the sentence (21) would be true also in case of Schliemann's not succeeding in locating the place (even if he were standing on the place, but not knowing about any connection of this place with Troja), and also in case of Troja's non-existence.

But we can also look for something else than just a location of an object. We can look for a murderer of somebody, etc. In this case we have a cognitive attitude to an office, we *want to know who* … . For instance since some date

(22) George IV. looked for the author of *Waverley*,

and though in the meantime he met Walter Scott several times, he ignored him till he found out the connection between the man and the novel. It means that *wishing to know who* (and similarly seeking in (20), (21)) does not express a relation of an individual "directly" to an individual. This kind of seeking might be a relation of an individual to an individual "indirectly via" an individual office (*de re*), or directly to the whole individual office (*de* dicto), or to the construction of the office.

In case of attitudes to *empirical notions* (unlike to mathematical notions) the latter possibility may be too restrictive. If somebody is seeking the murderer of his father and his brother then it would not follow that he is seeking the murderer of his brother and father.

If it were a relation to the value of the office (*de* re case) – which might at first sight seem to be the best choice, then the sentence were reasonable (had a truth-value) only in case of the existence of the occupant of the office. This generally does not have to hold in seeking contexts. We can, e.g., look for the motive of an X's act though it was an unintentional act, the person X acted without a motive. Thus *seeking* would be sometimes a relation to the value of the office (*de re*), which would demand the existence of the value, or other times to the whole office (*de dicto*), which would not demand the existence of the value in a world/time. But we may not have any knowledge on the existence of the value of an office, and this fact should not influence the semantics of the expression *seeking.* Hence in both cases the plausible solution is an explication of seeking as a relation to an office. After all, we arrived at the same conclusion also when explicating the above *seeking the location of.*

Finding

We may seek something without succeeding in finding that. We may also find something without (previous) looking for it, but this is a "chance met finding" that totally

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[19] The expression "Trója" could be analysed as denoting an individual of type ι ("Trója" would be considered to be a proper name), but in such a case we would presuppose a satisfied supposition of a time/space existence. If we put a question against *Ílias* in this sense, the expression "Trója" would be a name of a fictive individual and play rather a role of a free variable. Then the sentence "Troja never existed" might be paraphrased as: If Troja has attributes $P_1, P_2, \ldots, P_n$, then nothing is identical with Troja [Tichý 1988, 270]. If the attributes $P_1, P_2, \ldots, P_n$ are considered to be requisites of the *office* Trója, then "Trója" denotes an object of type $\iota_{\tau\omega}$ and the presupposition of (time/space) existence is not necessary. In this case, however, the rigidity of designating disappears – the office of Troja can be occupied by different individuals in different world/times. This rather a counterintuitive consequence can be avoided if we distinguish abstract individuals from particulars in space/time, and pragmatic *de re / de dicto* supposition of proper names [Gahér 1999]. We will not deal with this problem. Hence our preferred analysis is open to the possibility of non-existence of Troja, whereas the former were not.

differs from "intended finding". We will deal with the latter. In this case we may also wish to find something that actually does not exist. Now we seemingly again have two possibilities of assigning a type to F(inding):

a) F as a relation of an individual to a concrete place – location / $(o\iota\mu)_{\tau\omega}$;

b) F as a relation of an individual to the "location office" / $(o\iota\mu_{\tau\omega})_{\tau\omega}$;

If we attached to the case a), we would exclude those cases of (non-)finding when the existence of the location is not ensured. On the other hand we can infer the existence of the place from the fact of a successful finding. In case of the sentence:

(22) Schliemann found (discovered) Troja

it seems that the finding attitude (unlike seeking attitude) is an attitude to an individual [Jespersen 1999]. But Schliemann did not find the very individual Troja – he had to have known its identity in some way, otherwise he would not have known what he had been looking for; thus he rather discovered the location of Troja. Hence we assume that finding is an attitude to the value of an office, this time a "location office".

Type-theoretical analysis: Sch(liemann) / ι, L(ocation of) / $(\mu\iota)_{\tau\omega}$, LT(location of Troja) / $\mu_{\tau\omega}$, F(inding) / $(o\iota\mu)_{\tau\omega}$ and T(roja) / $\iota_{\tau\omega}$.

(22*) $[\lambda w\lambda t\ [^0F_{wt}\ ^0Sch\ [\lambda w\lambda t\ [^0L_{wt}\ ^0T_{wt}]]_{wt}]]$,

and the subject of finding

$[\lambda w\lambda t\ [^0L_{wt}{}^0T_{wt}]]_{wt}$,

is the value of the "location office" LT, constructed by $[\lambda w\lambda t\ [^0L_{wt}{}^0T_{wt}]]$, in a given world/time, i.e. this construction occurs *de re*. The component ^0T is also in the *de re* supposition ("indices" *w,t* – speaker's perspective), and from (22*) it follows that there are two existential propositions, namely the existence of the location of Troja, as well as the existence of Troja.

Let us analogically analyse the sentence

(23) Walter Raleigh **did not** find Eldorádo,

Where WR(aleigh) / ι, L(ocation) / $(\mu\iota)_{\tau\omega}$, F(inding) / $(o\iota\mu)_{\tau\omega}$, LE(location of Eldorado) / $\mu_{\tau\omega}$ and E(ldorádo) / $\iota_{\tau\omega}$. The sense of the sentence is as follows:

(23*) $[\lambda w\lambda t\ [^0\neg[^0F_{wt}\ ^0WR\ [\lambda w\lambda t\ [^0L_{wt}\ ^0E_{wt}]]_{wt}]]]$.

The component – construction of the LE office - $[\lambda w\lambda t\ [^0L_{wt}\ ^0E_{wt}]]$ occurs again *de re*. But since the office E does not have any value in the actual world/time, the composition $[^0M_{wt}{}^0E_{wt}]$ is *v*-improper and the LE office does not have any value as well, it is an undefined function (in the actual world/time). Simply, neither Eldorado nor its location exists. We get the conclusion that the proposition constructed by (23*) does not have any truth-value (in the actual world now). It does not seem to be in accordance with our intuition; we would certainly say that (23) is a fact, that it is true. How to solve this discrepancy?

If we assume that the semantics of the expression "(intend) to find" does not depend on empirical facts, then a solution will be attaching to the case b): generally to explicate finding as an attitude to an office constructed by a *de dicto* construction. Thus the sense of sentence (23) will be analysed by a construction, in which the constituent $[\lambda w\lambda t[^0L_{wt}{}^0E_{wt}]]$ occurs *de dicto*, i.e.

(23**) $[\lambda w \lambda t\ [^0\neg[^0F_{wt}\ ^0WR\ [\lambda w \lambda t\ [^0L_{wt}\ ^0E_{wt}]]]]]$,
and the whole construction constructs now a true proposition. The analysis of (22) should be adjusted in a similar way, which is in accordance to the fact that Schliemann intended to find location of Troja in such a situation when its existence where not guaranteed at all. A contingent empirical fact – successful discovery location of Troja – cannot influence the semantics of the expression "(intend) to find".

8. Anaphora and an existential presupposition.

Sentence (23) can be paraphrased:

(23′) It is not true that Walter Raleigh found the place where Eldorádo is.

Using anaphora and an "accommodated" reading of (23'), we obtain:

(24) Walter Raleigh looked for Eldorádo but he did not find **it**,

which can again be paraphrased (in accordance with the above analysis):

(24′) Walter Raleigh looked for the location of Eldorado and he did not find **it**.

Even if sentence (24) were in the scope of explicative power of extensionalistic theories of anaphora, the sentence (24') is not. This is also obvious from the following concatenation of sentences:

(25*) Walter Raleigh looked for Eldorádo. Eldorádo does not exist. Walter Raleigh could not find it (did not find).

9. Anaphora and attitudes to constructions (concepts)

In the sentence
(27) The Pope is happy and Charles knows **it**
the pronoun "it" refers (according to the above analysis of attitudes) to a proposition or to a propositional construction. Since we are dealing with an attitude to empirical sentence, we can consider an implicit attitude to a proposition, i.e. the construction $[\lambda w \lambda t\ [^0H_{wt}\ ^0P_{wt}]]$ will be used without trivialisation and it occurs *de dicto.* On the other hand a propositional connective has to be applied to truth-values not to propositions (or their constructions), hence both the propositions denoted by components of the conjunction are used (*de re* case):

(27*) $\lambda w \lambda t\ ([\lambda w \lambda t\ [^0H_{wt}\ ^0P_{wt}]]_{wt} \wedge \underbrace{[\lambda w \lambda t\ [\ ^0K_{wt}\ ^0Ch\ \underbrace{[\lambda w \lambda t\ [^0H_{wt}\ ^0P_{wt}]]}_{de\ dicto}\ ]]_{wt}}_{de\ re})$,

where K(now that...) is $(o\iota(o_{\tau\omega}))_{\tau\omega}$-object.

Hence the sentence
(28) Charles knows that the Pope is happy
expresses an attitude to empirical sentence, and we analyse it as an implicit attitude to a proposition, i.e. the construction $[\lambda w \lambda t\ [^0H_{wt}\ ^0P_{wt}]]$ is used without trivialisation and it occurs *de dicto*:

(28*) $[\lambda w\lambda t\ [{}^0K_{wt}\ {}^0Ch_{wt}\ [\lambda w\lambda t\ [{}^0H_{wt}\ {}^0P_{wt}]]]]$.

On the other hand the sentence

(29) Charles knows that 5 + 7 = 12

does not express an attitude to the truth-value True but to the construction of it (K(now)/$(o\iota{*}_1)_{\tau\omega}$, 5, 7/$\tau$, +/$(\tau\tau\tau)$, =/$(o\tau\tau)$)

(29*) $[\lambda w\lambda t\ [{}^0K_{wt}\ {}^0Ch_{wt}\ \mathbf{{}^0}[{}^0{=}\ [{}^0{+}\ {}^05\ {}^07]\ {}^012]]]]$,

which is captured by the trivialisation of the construction $[{}^0{=}\ [{}^0{+}\ {}^05\ {}^07]\ {}^012]$ and emphasised by bold symbol for trivialisation.

Will such an ambiguity concerning the subject of the intentional attitude lead to an ambiguity of the anaphoric connector?
In the sentence
(30) 5 + 7 = 12 and Charles knows it
the anaphoric connector *it* represents a function that associates the position, which is in the whole construction denoted by ‘it’, with a construction expressed by the antecedent component of the conjunction. Since this is a mathematical fact, knowing here must be an explicit attitude to the construction, for Charles’ knowing concerns the way of identifying mathematical truth not the truth itself. Otherwise Charles would be mathematically omniscient, when knowing the truth 5 + 7 = 12 who would know all the other mathematical truths as well.

Analysing type-theoretically – Ch/ι, K(now)/$(o\iota{*}_1)_{\tau\omega}$, 5, 7/$\tau$, +/$(\tau\tau\tau)$, =/$(o\tau\tau)$ – we get:

$$(30^*)\ \ [\lambda w\lambda t\ (\ [{}^0{=}\ [{}^0{+}\ {}^05\ {}^07]\ {}^012]_{wt} \wedge [\lambda w\lambda t\ \underbrace{[{}^0K_{wt}\ {}^0MCh_{wt}\ \mathbf{{}^0}\underbrace{[{}^0{=}\ [{}^0{+}\ {}^05\ {}^07]\ {}^012]}_{de\ dicto}]}_{de\ re}{}_{wt})\ ],$$

The explication of an anaphoric expression functioning in the context of attitudes would thus be “constant”: The object it refers to is a construction occurring *de dicto.* If the construction constructs a proposition, we have an implicit attitude to an intension. If it constructs a mathematical fact, we have an explicit attitude to the construction. The result of anaphoric referring can be embedded in particular “outer” suppositions.
Thus a “constant” role of anaphoric expressions can be used to explain the anaphoric phenomenon even in extensional contexts without extending the ambiguity of the subject of an attitude in case of one and the same attitude expression.

11. Summary

1. Anaphora in the case of mathematical attitudes
Anaphora is a choice function that associates the position indicated by the referring pronoun **(it)** with the first to the left (in the linear representation) subconstruction of the whole construction, denoted by the text, which fulfils all the type conditions specified by the meaning of **it** in its position. Referring expression can be a component of an expression that is in the *de re* suppositiom.

2. Anaphora in the case of empirical *de dicto* attitudes to an intension.
Anaphora is a choice function that associates the position indicated by the referring pronoun **(it)** with the intension that is identified by the first to the left (in the linear representation) subconstruction of the whole construction denoted by the text, which fulfils all the type conditions specified by the meaning of **it** in its position. Referring expression can be a component of an expression that is in the *de re* suppositiom.

3. Anaphora in the case of empirical *de re* (notional@) attitudes to an intension.
Anaphora is a choice function that associates the position indicated by the referring pronoun **(it)** with the value (in a given world/time) of the intension that is identified by the first to the left (in the linear representation) subconstruction of the whole construction denoted by the text, which fulfils all the type conditions specified by the meaning of **it** in its position.
Cases 2. and 3. can be extended and applied also to pragmatic expressions.

4. Anaphora in extensional contexts is a choice function that associates the position indicated by the referring pronoun **(it)** with the extension that is denoted by the first to the left (in the linear representation) subexpression of the whole text, which fulfils all the type conditions specified by the meaning of **it** in its position. This case is probably a rare one with the exception of referring to objects named by proper names. It concerns referring to objects (denoted by extensional expressions) like sets of individuals, sets of ordered tuples of individuals, etc.

Definition 3. (***Anaphora as a choice function***)
For the case of attitudes to constructions:
Anaphoric expression is the name of a choice function *f*, which associates position ξ indicated by this anaphoric expression in the whole construction with the construction IC, i.e. f: $\xi \rightarrow$ IC, where C is a subconstruction of D, $^0[^0\text{At}\ ^0\text{I}\ \xi]$ is a subconstruction D and the construction $^0[^0\text{At}\ ^0\text{Per}\ ^0\text{C}]$, which is as the above except that ^{0}C is substituted for ξ, is defined, I is a singulariser, C is a construction of type $\boldsymbol{*_n}$, D is the construction expressed by the text (the sentence or its component), At is a notional attitude, Per is an individual (a man) and position ξ indicated by the anaphoric expression is actually a variable of the selective function, which does not have its own name in a natural language distinct from the name of the function. If $[^0\text{At}\ ^0\text{Per}\ ^0\text{C}]$ is improper but type-theoretically correct, the whole expression is reasonable but does not have a value. If it is not type-theoretically correct, the whole expression does not have any sense.

We can deduce at least three simpler variants of the above definition:
In the case of referring to empirical sentences or notions, when the functional value of the denoted intensions does not play any role (the existential presupposition does not have to be fulfilled), the anaphora is a choice function the value of which is an intension constructed by a *de dicto* construction: This has been the case of sentence (27), and also of sentence (24):
(24′) Walter Raleigh sought the location of Eldorado and he did not find **it.**
Its sense is:

(24′*) $[\lambda w \lambda t\ ([\lambda w \lambda t [^0\text{S}_{wt}\ ^0\text{WR}\ [\lambda w \lambda t\ [^0\text{L}_{wt}\ ^0\text{E}_{wt}]]]]_{wt} \wedge$
$[\lambda w \lambda t\ [^0\neg[^0\text{F}_{wt}\ ^0\text{WR}\ [\lambda w \lambda t\ [^0\text{L}_{wt}\ ^0\text{E}_{wt}]]]]_{wt})]$.

Referred intensions do not have to be identified only by constructions of the first order, of course; the definition of anaphora is thus rather a schema of a definition that can be adjusted for every order.

If the object referred to by an anaphoric expression is an extension, then anaphora is a selective function the value of which is an extension. This would be the case, for instance, if we simply conceived seeking the location of an individual as seeking an individual:

(32) Schliemann sought Troja and he found **it.**

(32∗) $[\lambda w\lambda t\ ([\lambda w\lambda t[^0S_{wt}\ ^0Sch\ ^0T]]_{wt} \wedge [\lambda w\lambda t\ [^0F_{wt}\ ^0Sch\ ^0T]]_{wt})]$.

The sentence

(33) If somebody has been born at the rise of Sirius, then he will not die at the sea

does not have to be analysed in a very complicated way, as Neale [1990, 241] proposed:

(33_N) [every x: man x] (x is born at the rise of Sirius) →
[anybody x: man $x \wedge x$ is born at the rise of Sirius] (x will not die at the sea),

but:

(33_{TIL}) $[\lambda w\lambda t\ \forall x\ [^0BS_{wt}\ x \rightarrow \neg{}^0DS_{wt}\ x]]$,

where BS is the property of being born at the rise of Sirius and DS is the property of dying at the sea, and the constructions of (expressions denoting) the both properties are in the *de re* supposition. A standard 1st order predicate logic formula for a general statement corresponds to it (without variables for possible worlds / times):

(33_{PL}) $(\forall x)(BS(x) \rightarrow \neg DS(x))$.

In this case it means that the TIL solution is much closer to the standard one than the neorussellian[20] and also more intuitive.

Since generally attitudes can be iterated, the situation can be more complicated when supposition and perspectives of a speaker and believers (individuals to whom attitudes are ascribed) are combined, as, e.g., in the sentence:

(34) John knows that the Pope is happy and Charles believes **it.**

This requires further research and work on the problem of anaphora.

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[20] It is known, that by means of the lambda operator we can defind in λ-calcule all other operators of the kind "most", "less" , and we do not need follow Geach, but we can in many cases to accept standard explication of the anaphora phenomena, e.g. if indefinite pronoun *somebody* is antecedent to pronoun *he*: this is a representation of universal quantification.

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